Instituto Complutense de Análisis Económico

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I examine the evolution of contagion indexes between the European Financial sector and the sovereign sector (Austria, Belgium, France, Germany, Italy, Netherlands and Spain) during the European sovereign credit crisis. Contagion indexes, ΔCoV aR and ΔCoES, reflect events associated with extreme left tail returns and interdependencies between defaults different than those observed in tranquil times. These measures reveal very useful information concerning risk management. I use a copula approach with time-varying parameters to capture changes in the tail dependence between returns in the financial and the sovereign sectors. I employ a Switching Markov model to identify the most stressful moments of the contagion indicators. The results point out the emergence of Greek debt crisis on March 2010 and the vulnerable situation of Spain and Italy in summer 2011 as the main periods where the contagion from the sovereign to the financial sector was stronger. The decrease in contagion was gradual since the speech made by the ECB on July 26th; 2012. The statistical significance of the change in the contagion indicators is checked using boostrap tests.

**Keywords** CoVaR, Copula, European sovereign credit crisis, systemic risk

**JEL Classification** G18, G21, G32, G38

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# Contagion spillovers between sovereign and financial European sector from a $\Delta CoVaR$ approach. ☆

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## 1. Introduction

Systemic risk in biological terms is defined as a possible global disaster arising from the behaviour of a single individual of the species that coexist in the same environment. Likewise in Economics, systemic risk is the threat of a system breakdown because the effects of the interactions among individuals are undervalued, i.e. negative externalities arise from the relationship between economic agents. Besides, since systemic risk affects by nature all sectors, it should be evaluated not only within sectors, but also between sectors. The timely identification of spillovers between the financial and the sovereign sectors is a crucial topic for preventing scenarios such as the European sovereign credit crisis. Government guarantees

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and bailouts have helped to build a close relationship between the financial and sovereign sectors, ultimately triggering massive damages to the welfare state as well as political reactions in the form of populist movements along Europe. Sovereign debt positions held by banks in their portfolios and the link between the ratings of the financial and the sovereign sectors worked as transmission channels for risk from the sovereign to the financial sector. This two-way feedback, as named by Acharya et al. (2014), can become an adverse feedback loop between sectors in case of crisis. While such relationship was extensively studied during the European sovereign credit crisis (Albertazzi et al. 2014, Panetta et al. 2011, Acharya et al. 2014, Gray et al. 2007, Gerlach et al. 2010, Ejsing and Lemke 2011, Dieckmann and Plank 2011), how this loop has been weakened has not been so widely studied. This article studies how the credit risk contagion between these sectors has evolved during the period $2009-2016$.

To shed light on this matter we have to clarify first what is understood by contagion. There is not an unique criterion to identify a contagion event. Contagion is a sophisticated and multidimensional concept that has several features. The focus on a certain set of contagion characteristic will lead to a different methodology for building the contagion indicator. For instance, defining contagion as the spread of idiosyncratic negative shocks to other institutions may lead to a Vector Autoregression (VAR) framework. Indeed, most research on this topic has been conducted in a VAR framework (Alter and Beyer (2012), Bicu and Candelon (2012), Kok and Gross (2013), Alter and Schüler (2012), Chudik and Fratzscher (2012), Candelon et al. (2011)). Following the VAR methodology, impulse response functions and variance decomposition are employed to evaluate contagion through the effects of an idiosyncratic shock on the other economic agents. The contagion measure under this approach expresses mean effects, but a measure based on the left tail returns would be more useful for risk management proposes. Moreover, not all the dependence between the sovereign and the financial sectors should be considered contagion. They are not independent sectors and a certain level of connection may be advantageous. These two points, i.e. the behaviour in an adverse scenario and the interdependencies between sectors different than those observed in normal times, are the key features that lead in this paper to a different proposal of contagion measure. Certainly, the proposed contagion indicators have implications for investors, who need a risk management tool to assess the exposure of their sectoral portfolios from undesired links with other sectors which are not taken into account by unconditional risk measures such as Value-at-Risk ($VaR$) and Expected Shortfall ($ES$). Looking at the change in the conditional risk measures ($CoVaR$ and $CoES$) when the conditioning sector moves from normal times to a distress scenario gives essential information concerning the capital shortfall in the conditioned institution due to the existence of dependence between sectors i.e. $\Delta CoVaR$ and $\Delta CoES$. I employ $\Delta CoVaR$ and $\Delta CoES$ as indicators of credit risk contagion between the sovereign sector on a country level and the European financial sector as a whole. The financial sector is measured on an European level due to the cross positions of sovereign debt by European financial institutions and also because of the high level of integration in the Eurozone financial sector.

The methodological approach for building $\Delta CoVaR$ and $\Delta CoES$ should be flexible in order to characterize accurately marginal features such as heteroskedasticity, leverage effects, asymmetry and skewness, apart from considering different possible joint distributions and changes in dependence. A copula methodology where the copula parameter is time-varying combined with a suitable marginal model meets these criteria. This methodology is employed not only because of its straightforward decomposition of the joint distribution, but also due to computational reasons, being less time expensive than other approaches that imply numerical integration such as the GARCH proposal by Girardi and Ergün (2013). Once the contagion risk indicators have been built, I identify regimes for the level of contagion risk based on a Switching Markov model. In particular, the Switching Markov model points out that for most sovereign sectors the contagion to the financial sector was concentrated in two periods, a first one around March 2010, when the Greek debt crisis emerged and a second one in the summer of 2011 due to the confidence crisis that Spanish and Italian sovereign sectors were raised. Besides, the contagion from the financial sector to the sovereign sector seems to end later and more slowly than the contagion from the sovereign sector to the financial sector. Considering Draghi's speech on July $26^{th}$, 2012 as a breakpoint in the European credit crisis, I test a possible change in the distribution and in the mean of the contagion measures orthogonalized by its own past using a boostraping procedure. Results show a decrease in the mean level of contagion after the ECB's speech and a smaller downside spillover between sectors after the breakpoint compared to the period before July $26^{th}$, 2012.

The remainder of the article is organised as follows: Section 2 describes the framework where the contagion risk measures are applied. Section 3 suggests the copula approach for assessing $\Delta CoVaR$ and $\Delta CoES$, describing the different dependence structures considered in the paper. Section 4 presents the data employed for the empirical application. Section 5 shows the main results and robustness checks. Section 6 closes describing possible future research lines and some policy recommendations, pointing out $\Delta CoVaR$ and $\Delta CoES$ as suitable tools for assessing contagion from a risk management point of view.

## 2. Background

$CoVaR$ measure was introduced by Adrian and Brunnermeier (2016) as a systemic risk measure for identifying Systemically Important Financial Institutions (SIFIs). The aim was to express the minimum returns for the conditioned institution $y$ with some confidence level $(1-\beta)100\%$ given a quantile $\alpha$ of the returns distribution for conditioning institution $x$, i.e.,

$$P_{t-1}[r_{y,t} \leq CoVaR_{y|x,t}(\alpha,\beta)|r_{x,t} = VaR_{x,t}(\alpha)] = \beta. \tag{1}$$

Girardi and Ergün (2013) enhances $CoVaR$ definition to allow backtesting and improve the behaviour of $CoVaR$ as a function of the dependence between institutions (Mainik and Schaanning (2014), Zhang (2015)). The modified $CoVaR$ definition expresses the minimum returns for the conditioned institution $y$ with some confidence level $(1-\beta)100\%$ given that the conditioning institution $x$ is below its $\alpha 100\%$ worst case scenario, i.e.

$$P_{t-1}[r_{y,t} \leq CoVaR_{y|x,t}(\alpha,\beta)|r_{x,t} \leq VaR_{x,t}(\alpha)] = \beta. \tag{2}$$

I employ the subscript $f$ for representing the global European financial sector and $s$ for the European sovereign sectors. The level $\alpha$ of the conditioning event is usually fixed at $\alpha = \beta$ where $\alpha, \beta \in (0, 1)$, and due to the focus on the left tail returns, $\alpha$ and $\beta$ are close to zero in a distress scenario. Employing a conditioning event as the $VaR$, which is independent of the level of risk of the conditioning institution $x$, allows us to compare $CoVaR$ given several conditioning institutions with different risk profiles.

Even though the $CoVaR$ properties improve under Equation (2), it still has some limitations since it looks only to a certain percentile of the conditioned institution $y$ and consequently it is not subadditive. This feature can be enhanced if the Value-at-Risk dimension is moved to an Expected Shortfall framework. The Conditional Expected Shortfall, $CoES_{y|x,t}(\alpha, \beta)$, measures the average return for institution $y$ when the returns are lower than $CoVaR_{y|x,t}(\alpha, \beta)$, i.e.

$$CoES_{y|x,t}(\alpha, \beta) = \frac{1}{\beta} \int_0^{\beta} CoVaR_{y|x,t}(\alpha, q)dq, \tag{3}$$

where $CoVaR_{y|x,t}(\alpha, q)$ is given by Equation (2).

Losses not considered in normal scenarios can trigger out a systemic event because of lack of liquidity, i.e. in a normal scenario capital needs can be fulfilled without spillover effect between sectors, but in a distress scenario capital needs could lead to bankrupt and bailout processes, triggering out a contagion event between the sovereign and the financial sectors. Therefore, $CoVaR$ and $CoES$ are unsatisfactory measures for assessing the contagion between sectors. Indeed, they may be enough to capture the losses in a given scenario but not the loss changes when the conditioning scenario moves. The change in the previous conditional risk measures when the conditioning variable moves from tranquil times to a distress period are known as Delta Conditional measures, i.e. $\Delta CoVaR$ and $\Delta CoES$. There is no consensus about the definition of tranquil times under $\Delta CoVaR$. Chen and Khashanah (2014) employs the unconditional $VaR$ measure and Girardi and Ergün (2013) uses a standard deviation range around the mean value of the conditioning variable. However, the former definition responds to the importance of taking into account the conditioning variable for risk assessment proposed but it does not capture the relevance of a change in the conditioning variable from a normal period to a distress scenario for the conditioned variable. On the other hand, the latter definition for normal scenario is not fully defined for non-Gaussian marginal distributions due to the need to use higher moments, e.g. skewness and kurtosis. In this article, the normal scenario is defined as a $\beta/2$ range of quantiles around the median. Consequently, I define $\Delta CoVaR_{y|x,t}(\beta)$ as

$$\Delta CoVaR_{y|x,t}(\beta) = CoVaR_{y|x,t}(\alpha_s, \beta) - CoVaR_{y|x,t}(\alpha_n, \beta), \tag{4}$$

where $\alpha_s = \beta$ in Equation (2) , i.e.

$$P_{t-1}[r_{y,t} \leq CoVaR_{y|x,t}(\alpha_s, \beta) | r_{x,t} \leq VaR_{x,t}(\beta)] = \beta$$

and $\alpha_n$ are the set of quantiles between the lower percentile $\alpha^-$ and the upper percentile $\alpha^+$ such that

$$P_{t-1}[r_{y,t} \leq CoVaR_{y|x,t}(\alpha_n, \beta) | VaR_{x,t}(\alpha^-) \leq r_{x,t} \leq VaR_{x,t}(\alpha^+)] = \beta,$$

where $\alpha^+ = 0.5 + \beta/2$ and $\alpha^- = 0.5 - \beta/2$. The idea of considering an upper and a lower bound for the conditioning variable was already considered by Reboredo and Ugolini (2016). The proposed definition for a normal scenario is as accurate as the one for the distress scenario, because we are considering the same $\beta$ range of quantiles, and it is fully defined in percentile terms. These features were not fulfilled by previous definitions. $\Delta CoVaR_{y|x,t}(\beta)$ expresses the undervaluation of the minimum returns measure with a confidence level $(1-\beta)100\%$ for institution $y$ when institution $x$ moves from normal times to an adverse scenario. $\Delta CoES$ can be computed following the same procedure as in Equation (4).

Delta Conditional measures do not distinguish whether the increase in the risk measure is due to causal reasons or to a common factor between both institutions. Hence, I will capture changes in the conditioned institution even in the absence of a direct link. Imagine that the financial sector $f$ has a diversified sovereign debt portofolio where an isolated bankrupt in one country $s$ would not cause contagion to the financial system. However, Delta Conditional measures would disclose contagion if the distress is due to a common factor of the set of countries. Although $\Delta CoVaR$ and $\Delta CoES$ do not express causality, they are directional measures, i.e. $\Delta CoVaR_{f|s,t} \neq \Delta CoVaR_{s|f,t}$.

## 3. Methodology

The model structure for $CoVaR$ can be divided into three steps: the marginal model structure that gathers individual features as heterokedasticity or kurtosis, the copula function that links marginal density functions and the copula time-varying parameter that allows changes in tail dependence. The assessment of $CoVaR$ is straightforward given these three stages.

Following Bayes' theorem and copula theory Equation (2) can be rewritten as a ratio of probabilities, i.e.

$$\begin{aligned} P_{t-1}[r_{y,t} \leq CoVaR_{y|x,t}(\alpha, \beta) | r_{x,t} \leq VaR_{x,t}(\alpha)] &= \frac{C(u_y, \alpha; \theta_t)}{\alpha} \\ &= \beta, \end{aligned}$$

where $\theta_t$ is the copula parameter at time $t$, $P_{t-1}[r_{y,t} \leq CoVaR_{y|x,t}(\alpha, \beta), r_{x,t} \leq VaR_{x,t}(\alpha)] = C(u_y, \alpha; \theta_t)$ and $P_{t-1}[r_{x,t} \leq VaR_{x,t}(\alpha)] = \alpha$.

Therefore $CoVaR_{y|x,t}(\alpha, \beta)$ is obtained by identifying the value $u_y^*$ such that $C(u_y^*, \alpha) = \alpha\beta$ and then employing the inverse cumulative distribution function of institution $y$'s returns, i.e. $F^{-1}_{r_{y,t}}(u_y*) = CoVaR_{y|x,t}(\alpha, \beta)$.

In this section, first I describe the marginal model and the assumption distribution about the innovation, then I present a set of copulas considered to model the joint distribution between the financial and the sovereign sector and finally I establish the dynamic evolution in the copula parameter to allow time-varying dependence between sectors.

### *3.1. Marginal model*

For each sector, I estimate a P-order autoregressive model ($AR(P)$) where the lag $P$, for parsimony reasons, is the minimum such that the innovation has no autocorrelation. I also model heterokedasticity and leverage effect by using a GJR-GARCH(1,1) representation. Finally I model skewness and kurtosis by assuming a Hansen (1994)'s skewed t distribution for the innovations. That is,

$$r_{j,t} = \underbrace{\phi_{j,0} + \sum_{k=1}^{P} \phi_{j,1} r_{j,t-k}}_{\mu_{j,t}} + \varepsilon_{j,t}, \qquad j = f, s \tag{5}$$

with $\varepsilon_{j,t} = \sigma_{j,t}\xi_{j,t}$ where $E\left(\varepsilon_{j,t}\varepsilon_{j,t-k}\right) = 0$ for $\forall k > 0$ and $\sigma^2_{j,t}$ is the conditional variance given by a GJR-GARCH(1,1) specification, i.e.

$$\sigma^2_{j,t} = \omega_i + \alpha_j(1 + \theta_j \mathbb{1}_{j,t-1})\varepsilon^2_{j,t} + \beta_j \sigma^2_{j,t-1}, \tag{6}$$

where the indicator function $\mathbb{1}_{j,t-1}$ values 1 if $\epsilon_{j,t} < 0$ and zero otherwise and $\xi_{j,t} \sim f(\xi_{j,t}; \eta_j, \lambda_j)$ where $f$ is the probability distribution function of the skewed-t distribution, $\eta_j$ denotes the number of degrees of freedom and $\lambda_j$ the asymmetry parameter.
The density of Hansen (1994)'s skewed-t distribution is

$$h(\xi_t|\eta,\lambda) = \begin{cases} bc(1 + \frac{1}{\eta-2}(\frac{b\xi_t + a}{1-\lambda})^2)^{-(\eta+1)/2} & \xi_t < -a/b \\ bc(1 + \frac{1}{\eta-2}(\frac{b\xi_t + a}{1+\lambda})^2)^{-(\eta+1)/2} & \xi_t \geq -a/b \end{cases}, \tag{7}$$

where $2 < \eta < \infty$ and $-1 < \lambda < 1$. The constants $a$, $b$ and $c$ are given by

$$a = 4c\lambda\left(\frac{\eta-2}{\eta-1}\right), b = \sqrt{1 + 3\lambda^2 - a^2}, c = \frac{\Gamma(\frac{\eta+1}{2})}{\sqrt{\pi(\eta-2)}\Gamma(\frac{\eta}{2})}.$$

Note that when $\lambda = 0$ Equation (7) reduces to the standard Gaussian distribution as $\eta \to \infty$. When $\lambda = 0$ and $\eta$ finite, we obtain the standardized symmetric-t distribution.

### *3.2. Copula function*

The choice of copula determines the relationship between a couple of marginal distributions. An inaccurate copula choice would suppose missleading $CoVaR$ and $\Delta CoVaR$ estimates and ultimately a wrong interpretation of their values. To diminish that chance I compare a broad range of copula choices using Akaike Information Criterion ($AICC$) corrected for small sample bias as suggested by Hurvich and Tsai (1989). $AICC$ criterion has been employed for copula selection in other $CoVaR$ studies such as Reboredo and Ugolini (2015b). I consider 8 alternative copulas that are broadly employed in financial studies. Each copula implies a different tail dependence. The Clayton and the Survival Gumbel copulas allow for lower tail dependence but no upper tail dependence, whereas the opposite situation is found in Gumbel copula. The Joe-Clayton (BB7), the Student t and the Clayton-Gumbel (BB1) copulas allow for either upper and lower tail dependence. Table D.1 presents the main tail dependence features in the set of employed copulas.

**[Insert Table D.1 here]**

*3.3. Time evolution in the copula parameter*

I assume that the functional form of the copula remains fixed over the sample while the parameters for each copula are varying based on some equation for the time evolution. A time-varying copula parameter allows changes in tail dependence along time. As a result, the model is more flexible for tracking changes in the relationship between both sectors. I employ the approach proposed by Patton (2006). Alternative approaches for modeling time-varying copula parameter may be using rolling-windows (Aloui et al. (2013)), Generalized Autoregressive Score ($GAS$) (Creal et al. (2013)) or Stochastic Autoregressive copulas ($SCAR$) (Hafner and Manner (2012)). Another way to have a change in tail dependence over time is to assume different states, each of one characterized by a certain copula, a regimen-switching copulas like Rodríguez (2007). A comparative analysis of them is out of the scope of this work.

The parametric representation for the Clayton, the Gumbel and the Survival Gumbel copulas is

$$\theta_t = \Lambda_1 \left( \omega + \beta \theta_{t-1} + \alpha \frac{1}{20} \sum_{k=1}^{20} |u_{s,t-k} - u_{f,t-k}| \right), \tag{8}$$

where $\Lambda_1$ is $\exp(x)$ for the Clayton copula and $(\exp(x) + 1)$ for the Gumbel and Survival Gumbel copula to keep the values in the feasible region of the parameter space. The evolution for the parameter $\delta$ of Frank copula is represented by

$$\delta_t = \omega + \beta \delta_{t-1} + \alpha \frac{1}{20} \sum_{k=1}^{20} |u_{s,t-k} - u_{f,t-k}|. \tag{9}$$

The evolution equation for the two parameter families of non-elliptical copulas , i.e. BB1 and BB7 copulas, is based on the link between these parameters and the tail dependence, which is disclosed in Table D.1.

$$\tau_t^K = \Lambda_2 \left( \omega_K + \beta_K \tau_{t-1}^K + \alpha_K \frac{1}{20} \sum_{k=1}^{20} |u_{s,t-k} - u_{f,t-k}| \right), \qquad K = U, L \tag{10}$$

where $\Lambda_2(x) \equiv (1 + \exp(-x))^{-1}$ is the logistic transformation to keep the tail dependence coefficient between 0 and 1.

For the Student t copula I assume that the number of degrees of freedom is constant (Elliott and Timmermann, 2013, p. 932 , Reboredo and Ugolini, 2016) and only the correlation parameter, i.e., $\rho_t$, is time-varying. The dynamics for the parameter of elliptical copulas is

$$\rho_t = \Lambda_3 \left( \omega + \beta \rho_{t-1} + \alpha \frac{1}{20} \sum_{k=1}^{20} \Phi^{-1}(u_{s,t-k}) \Phi^{-1}(u_{f,t-k}) \right),$$

where $\Phi^{-1}$ is either the inverse Gaussian cumulative distribution function in case the elliptical copula is Gaussian or the inverse Student t cumulative distribution function with $\eta$ degrees of freedom for the Student t copula. The modified logistic transformation allows for a value of $\rho_t \in (-1, 1)$, i.e. $\Lambda_3(x) \equiv \frac{1-\exp(-x)}{1+\exp(-x)}$.
Table D.2 provides a summary of the time-varying parameters representation proposed for each copula.

[**Insert Table D.2 here**]

The joint density function is obtained by combining the marginal probability distribution functions and the density copula function. I employ the two-step method of Inference Functions for Margins (IFM) to estimate the parameters by maximum log-likelihood, where marginal distributions and copulas are estimated separately. The computational cost of finding the optimal set of parameters is significantly reduced significantly by this approach. Joe and Xu (1996) shows that the estimated parameters using IFM method are consistent and asymptotically normal.

## 4. Data

It is widely accepted that the European sovereign debt crisis was led by a confidence crisis in the institutions. Consequently I employ a credit derivative, credit default swaps (CDS), obtained from Datastream on weekly basis from May $22^{th}$, 2009 to May $13^{th}$, 2016 to compute $CoVaR$ measure. The total number of observations is 338.
I use the 5-year contract because it is the most liquid maturity. Concerning the restructuring event, I choose complete restructuring, also known as old reestructuring because its credit event is used mainly in Europe and it is the usual one for sovereign institutions (Anson et al., 2004, p. 62). Moreover, the CDS employed in this study are those with a senior debt underlying since it is the most traded branch of the CDS categories. I choose the same type, seniority and maturity for the financial firms' CDS.
I consider sovereign CDS from Austria, Belgium, France, Germany, Italy, Netherlands and Spain. A total of 25 European bank CDS meet the criteria for the considered period, 14 being banks from the core European area whereas 11 are in the periphery. The number of banks and their countries are: Austria (2), Belgium (1), Finland (1), France (5), Germany (5), Italy (4), Netherland(3), Portugal (1) and Spain (3).

[**Insert Table D.3 here**]

I build financial CDS indices for each country by taking the median CDS spread in a given country each week. Later, I transform them into returns and I obtain the common financial risk factor among CDS spread using principal component analysis[1]. According to Rodríguez-Moreno and Peña (2013), the first principal component of a CDS portfolio is the best systemic measure in the macro group. Table D.4 shows the weight under the principal component analysis. To check for robustness, the equally weighted financial portfolio is also built with similar results.

[**Insert Table D.4 here**]

CDS spreads are transformed in returns following Berndt and Obreja (2010) and Ballester et al. (2016).

$$\begin{aligned} r_{i,t} &= -\Delta CDS_t A_t(T) \\ &= -\Delta CDS_t \frac{1}{4} \sum_{j=1}^{4T} \delta\left(t, \frac{j}{4}\right) q\left(t, \frac{j}{4}\right), \end{aligned} \tag{11}$$

where $\Delta CDS_t(T)$ is the weekly change in CDS spreads with maturity T and $A_t(T)$ is the value of a defaultable quarterly annuity over the next T years. $T$ is equal to five years, given the selected CDS data. The risk-free discount factor for day $t$ and $s$ quarter is $\delta(t,s)$, fitted from Euribor rates[2]. The risk-neutral survival probability of the bank or government over the next $s$ quarters can be written as $q(t,s) = \exp(-\lambda_t(s))$ where $\lambda_t$ is the risk-neutral default intensity. $\lambda_t$ is computed directly from observed CDS spreads as $\lambda_t = 4\log(1 + CDS_t/4L)$. $L$ denotes the risk neutral expected loss given default (LGD), fixed at 60% for corporate firms and 40% for governments. Note that the change in CDS spreads is used for returns estimation preceded by a minus sign, so an increase in credit risk, i.e. a raise in CDS spreads, supposes a decrease in CDS returns whereas a reduction of credit risk implies an increase in CDS returns. Figure C.1 shows the path of the CDS quote in basic points (green line) and the price employing an exponential function on the returns obtained from Equation (11) (blue line). Prices seems to react to the same shocks as CDS, although in opposite directions. The black line indicates the week of July $26^{th}$, 2012 when Mario Draghi made a speech that, as can be seen in the Figure, changed the trend for Spanish and Italian CDS quotes.

[**Insert Figure C.1 here**]

Table D.5 provides descriptive statistics for the CDS returns of the financial sector and the European countries. The excess kurtosis and the skewness in the data support the choice

[1] In order to avoid giving an excessive weight to the most volatile country-level CDS returns, the PCA is performed on the correlation matrix. This approach is similar to the one employed by Chamizo and Novales Cinca (2016) for obtaining the returns of the financial system credit risk.

[2] Euribor rates are obtained from the European Money Markets Institute (EMMI) and floored at 0%.

of the skewed t distribution for innovations. Annual volatility is around $45 - 50\%$, in line with volatility in the stock market. Furthermore, CDS returns from the financial sector have lower volatility than sovereign CDS returns and also the lowest mean return.

**[Insert Table D.5 here]**

## 5. Results

I discuss the results for contagion between the financial and the sovereign sector by presenting first the results of the marginal distribution from which I obtain the inputs for the copula and from which I assess the quantile for the conditioning variable. Later I discuss the results for the copula estimations and copula choice, from which I assess the conditional quantile. Finally I discuss the Delta Conditional measures ($\Delta CoVaR$ and $\Delta CoES$), finding stress periods for these indicators using a Switching Markov process. I employ boostrap tests to check for a possible change in the distribution.

### *5.1. Results for the marginal models*

Table D.6 shows the estimated parameters with the z-statistics in brackets. A first order autoregressive model is employed for all the sectors excepted for Spanish and Italian sovereign CDS returns. For these countries a second order autoregressive model is employed considering the autocorrelation analysis and the backtesting performance. Unconditional coverage backtesting test proposed by Kupiec (1995) and the conditional one proposed by Christoffersen (1998) are used for testing the number of exceedances of a $VaR$ with a 5% significance level. All the models pass the tests as show by their p-values, i.e., we do not reject neither that the probability of having an exceedance is a 5% nor that those exceedances are independent from each other. P-values for Ljung-Box and Engle's ARCH tests show that autocorrelation and heteroscedasticity are corrected gathered in the model.

**[Insert Table D.6 here]**

### *5.2. Results for the copula model*

I estimate different types of copulas (see Table D.1) using the skewed-t cumulative distribution function of the standardized residuals for each of the marginal models. Table D.7 and D.8 summarize the estimated parameters and the standard deviation between brackets.

**[Insert Table D.7 here]**

**[Insert Table D.8 here]**

The interpretation of those values is harder than in a GARCH model due to the transformation needed to keep the time-varying parameter in a region of the parameter space. However, the time-varying evolution of the copula parameter is plotted for the selected copulas according to the AICC criterion. Figure C.2 shows the time-series evolution of the copula parameter between the financial sector and each one of the sovereign sectors. Austria clearly presents a peak in dependence after May 2012, whereas the sovereign sectors of France and Belgium reduce significantly their dependence parameter with the financial sector at the end of the sample.

**[Insert Table D.9 here]**
**[Insert Figure C.2 here]**

The copula selection according to AICC criterion (see Table D.9) could lead to choose a copula that fits really well the higher tail of the joint distribution but not so well for lower quantiles. To double check the chosen copulas, Table D.10 presents backtesting results for the frequency of the exceedances below the 5% quantile of returns in conditioned institution $y$ when there are exceedances below the ex-ante $VaR_x(0.5)$ of the conditioning. This corresponds to $CoVaR_{y|x,t}(0.5, 0.05)$ in Equation (2). Table D.10 shows the p-values of the $CoVaR$ for the unconditional coverage test proposed by Kupiec (1995). Besides the p-values, the upper and lower bound of the non-rejection area with 5% significance level is presented jointly with the number of exceedances of the conditioned variable, i.e. the exceedance bounds out of which we could reject the null hypothesis with a 95% confidence level. The number of exceedances for the conditioning variable is also shown in Table D.10. Table D.10 shows the p-values for the conditional coverage test proposed by Christoffersen (1998) to detect possible clusters in the exceedances of the conditioned variable.
Both backtesting tests, i.e. unconditioned and conditioned, are passed with a 5% significance level in both directions, i.e. $CoVaR_{s|f,t}(0.5, 0.05)$ and $CoVaR_{f|s,t}(0.5, 0.05)$. Further information about how these tests are built can be found in Appendix A.

**[Insert Table D.10 here]**

*5.3. Contagion indicator results*

Figures C.3 and C.4 present weekly returns for the sovereign sector and the financial sector (in the blue lines), the Value-at-Risk assessed with a 95% confidence level (in the black lines), and the minuend and subtrahend from which is built the $\Delta CoVaR_{y|x}(\beta)$, i.e. the $VaR$ for the conditioned institution when the conditioning institution is in normal times or $CoVaR_{y,x,t}(\alpha_n, \beta)$ (in the magenta lines) and the $VaR$ for the conditioned institution when the conditioning institution is in distress or $CoVaR_{y,x,t}(\alpha_s, \beta)$ (in the red lines). All the risk measures are computed with a 95% confidence level, i.e., $\beta = 0.05$.

**[Insert Figure C.3 here]**
**[Insert Figure C.4 here]**

I employ a Switching Markov model which endogenously identifies periods of extreme contagion for $\Delta CoVaR$ measures. The assumption that time series properties of $\Delta CoVaR$, e.g. mean and variance, are state-dependent where the transition between states occurs stochastically allows us to distinguish periods of high contagion from periods of moderate contagion. In other words, the cumulative distribution function for each $\Delta CoVaR$ measure is approximated using a mixture of normal distributions where probabilities are given by a first order Markov Chain. The number of states assumed may be two or three because of the economical interpretation of low, medium or high contagion regime. The number of states and the changes in parameters, i.e. changes in the mean parameter or also in the variance parameter, are chosen according to the values of AICC and Regimen Classification Measure

(RCM) like Hollo et al. (2012). I choose the regime with the lowest mean $\Delta CoVaR$ as the distress period. Figures C.5 and C.6 show these periods of crises shaded for the contagion measures $\Delta CoVaR$ and $\Delta CoES$. Figures C.5 and C.6 show that are two main periods of stress, one around March 2010 when the Greek debt crisis arose and a second one around August 2011 when the ECB had to bought Italian and Spanish debt using the Securities Market Program. Black line represents July $26^{th}$, 2012 when Mario Draghi made the speech when he said that the ECB would do whatever it takes to preserve the Euro[3]. Note that distress periods after that date are less frequent and with a lower length than before, but according to the Switching Markov approach the contagion periods did not stop suddenly then for most of the countries. Moreover, the high contagion period from the sovereign sector to the financial sector seems to end before than vice versa.

**[Insert Figure C.5 here]**
**[Insert Figure C.6 here]**

I perform some boostrap tests to double check the change in the contagion risk indicators since July $26^{th}$, 2012. First, I delete autocorrelation in the $\Delta CoVaR$ time-series by orthogonalize it from previous month values, so the new time series can not be explained by the past trend of the indicator. The fact of estimate the parameters of $\Delta CoVaR$ introduces a nuisance parameter that invalidates the Kolmogorov-Smirnov (KS) test for equal distribution of the time-series before and after the breakpoint. Certainty, this nuisance parameter affects to the free-distribution of the KS test (Durbin, J. (1973)). Abadie (2002) proposes a boostrap KS test to deal with this problem. Bernal et al. (2014) and Reboredo and Ugolini (2015a) along others use this test in the $CoVaR$ framework. I employ the boostrap procedure for building also a mean test, obtaining the critical values through the boostrap method. Appendix B explains in detail the procedure for building these tests.
The null hypothesis of mean before July $26^{th}$, 2012 is lower or equal than after in absolute value is rejected at a 5% significance level for most contagion indexes. There is enough statistical evidence against the null hypothesis that the mean contagion levels between the financial and the sovereign sectors before July $26^{th}$, 2012 are lower or equal than after this breakpoint.

## 6. Conclusions

I have examined how contagion indexes between the European financial sector and sovereign sector in a country level (Austria, Belgium, France, Germany, Italy, Netherlands, Spain) evolved during the European sovereign credit crisis, before and after the breakpoint stated by the ECB's speech on July $26^{th}$, 2012. In this article $CoVaR$ measure is employed with a copula methodology to assess the interrelationship between sovereign and financial credit risk. The economic literature has not employed yet this approach to deal with the spillovers between sovereign and financial credit risk. This approach is a robust way of

[3]This date was chosen ad hoc due to the impact in the CDS quotes and in the stock markets

measuring systemic risk focusing on a low quantile of the returns' distribution.

The copula methodology allows us to decompose the joint distribution in an understandable way, besides of being a time-saving and less computationally expensive method than other procedures. The time-varying representation of the copula parameter allows for a flexible adaptation to the sample, capturing changes in tail dependence and contagion levels. Indeed, the copula parameter plays a key role weighting the level of stress for each assets, as measured by its volatility, by its dependence with the other sector. Future studies might try to find out how different dynamics for the time evolution of the copula parameter might affect to the numerical values of $\Delta CoVaR$ and $\Delta CoES$. These methodological improvements in the model could increase model risk due to the high adaptability of the model. The possibility of more flexibility could mean a noisy estimation.

Using weekly CDS returns from May 2009 to May 2016, a switching Markov model estimated for $\Delta CoVaR$ indicates two main periods of contagion between both sectors. The first period would be related to the surge of Greek problems on March 2010 while the second period in summer 2011 might be due to the doubts concerning Spain and Italy. The contagion from sovereign to financial crisis seems to almost finish after the ECB's speech on July $26^{th}$, 2012 . On the other hand, contagion from the financial to the sovereign sector seems to decrease more slowly. Several boostrap tests are computed to check if there was a change in contagion after this breakpoint. Results show a change in the level and range of values taken by $\Delta CoVaR$.

Policy makers need indicators for assessing the effectiveness and the collateral detrimental effects that some policy measures can have on the economy. The contagion indicators built using $CoVaR$ methodology have implications for investors, who need a risk management tool to assess the exposure of their sectoral portfolios from undesired links with other sectors which are not taken into by unconditional risk measures. For instance, this measure provides information with a certain confidence level on how much could increase the maximum loss from a sovereign debt portfolio if a financial crisis occurs. $\Delta CoVaR$ and $\Delta CoES$ provide support as tools for the measurement of contagion and spillover effects. Our findings reveal them as suitable tools to provide reliable information for taking effective and efficient policy measures.

**Acknowledgement**

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## Appendix A. Backtesting procedure on CoVaR

The proportion of exceedances over the threshold of the CoVaR should equal approximately the significance level and they should take place independently, not in clusters. Consequently to check the accuracy of the proposed model we can use the statistical tests for unconditional coverage from

Kupiec (1995) and the conditional coverage from Christoffersen (1998). The null hypothesis of the unconditional and conditional coverage is performed at 5% level of significance under skewed-t margins and the best fit according to the corrected Akaike Information Criterion (AICC).

For the conditioned institution in $CoVaR_{y|x}(\alpha, \beta)$ I build the indicator function that takes the value one if past ex-post losses of $x$ cross the ex-ante VaR forecast and zero otherwise, i.e.,

$$\mathbb{1}_{x,t} = \begin{cases} 1 & if \quad r_{x,t} \leq VaR_{x,t}(\alpha) \\ 0 & if \quad r_{x,t} > VaR_{x,t}(\alpha) \end{cases} .$$

For those days $t$ where $\mathbb{1}_{x,t} = 1$ I use a second indicator function that values one if past ex-post losses of $y$ cross the ex-ante $CoVaR$ forecast and zero otherwise, i.e.,

$$\mathbb{1}_{j|l,t} = \begin{cases} 1 & if \quad r_{y,t} \leq CoVaR_{y,t}(\alpha, \beta) \\ 0 & if \quad r_{y,t} > CoVaR_{y,t}(\alpha, \beta) \end{cases} .$$

For this last hit sequence I have $T_{\mathbb{1}_{x,t}=1}$ observations , i.e., the observations where $r_{x,t} \leq VaR_{x,t}$.Consequently, to build the backtesting procedure I only employ $T_{\mathbb{1}_{x,t}=1}$ observations and not all the sample as in the backtesting procedure on $VaR$.

**Unconditional coverage test from Kupiec (1995).** The proportion of exceedances over the threshold is equal to the significance level if $CoVaR_{y|x}(\alpha, \beta)$ satisfies the unconditional coverage property, i.e. $P(\mathbb{1}_{y|x,t+1} = 1) = \beta$. Consequently the null and alternative hypothesis in this test would be

$$\begin{cases} H_0 : E[\mathbb{1}_{y|x,t}] \equiv p = \beta, \\ H_1 : E[\mathbb{1}_{y|x,t}] \equiv p \neq \beta. \end{cases}$$

Let us define $X = \sum_{t=1}^{T_{\mathbb{1}_{x,t}=1}} \mathbb{1}_{y|x,t}$, then the likelihood ratio of Kupiec (1995) is given by

$$LR = \frac{p^X (1-p)^{T_{\mathbb{1}_{x,t}=1} - X}}{\left(\frac{T_{\mathbb{1}_{x,t}=1} - X}{T_{\mathbb{1}_{x,t}=1}}\right)^{T_{\mathbb{1}_{x,t}=1} - X} \left(\frac{X}{T_{\mathbb{1}_{x,t}=1}}\right)^X},$$

where $-2\log(LR) \sim \chi^2_1$ under the null hypothesis.

**Conditional coverage test from Christoffersen (1998).** The expected proportion of exceedances over the threshold at $t+1$ is independent to the proportion of exceedances at $t$ to satisfy the conditional coverage property, $P_t(\mathbb{1}_{y|x,t+1} = 1) = \beta$. Given the assumption that $\mathbb{1}_{x|l,t}$ follows a first-order Markov sequence with transition probability matrix

$$P_1 = \begin{bmatrix} 1 - p_{01} & p_{01} \\ 1 - p_{11} & p_{11} \end{bmatrix},$$

where $p_{k,q}$ indicate the probability of having in t+1 $\mathbb{1}_{y|x,t+1} = q$ conditional to the scenario on t where $\mathbb{1}_{y|x,t} = k$ with $q, k = 0, 1$. The probability of a exception in t+1 doesn't depend on the fact of having an exception on $t$ if the conditional coverage property is satisfied, i.e. $P_t(\mathbb{1}_{y|x,t+1} = 1) = P(\mathbb{1}_{y|x,t+1} = 1)$. In conclusion, the null and the alternative hypothesis are

$$\begin{cases} H_0 : E[\mathbb{1}_{y|x,t}] \equiv p = p_{01} = p_{11}, \\ H_1 : E[\mathbb{1}_{y|x,t}] \equiv p \neq p_{01} = p_{11}, \end{cases}$$

Given the fact that there are $T_{\mathbb{1}_{x,t}=1}$ observations, a total of $T^{pair}_{\mathbb{1}_{x,t}=1} \equiv T_{\mathbb{1}_{x,t}=1} - 1$ pair of observations can be obtained. The sample of pair of observations can be divided in four subsamples, i.e.

$$T^{pair}_{\mathbb{1}_{x,t}=1} = T^{pair,00}_{\mathbb{1}_{x,t}=1} + T^{pair,01}_{\mathbb{1}_{x,t}=1} + T^{pair,10}_{\mathbb{1}_{x,t}=1} + T^{pair,11}_{\mathbb{1}_{x,t}=1},$$

where the superscripts indicate that if there was an exceedance at periods $t$ and $t-1$ and the subscript indicate that all the observations hold $r_{x,t+1} \leq VaR_{x,t+1}$.
Defining

$$\hat{p}_{01} = \frac{T^{pair,01}_{\mathbb{1}_{x,t}=1}}{T^{pair,00}_{\mathbb{1}_{x,t}=1} + T^{pair,01}_{\mathbb{1}_{x,t}=1}},$$

and

$$\hat{p}_{11} = \frac{T^{pair,11}_{\mathbb{1}_{x,t}=1}}{T^{pair,10}_{\mathbb{1}_{x,t}=1} + T^{pair,11}_{\mathbb{1}_{x,t}=1}},$$

$H_0$ holds if $\hat{p}_{01} \approx \hat{p}_{11}$, as a consequence the probability of having an exceedance in $t+1$ could be defined without taking into account the scenario in $t$, i.e.,

$$\hat{p} = \frac{T^{pair,01}_{\mathbb{1}_{x,t}=1} + T^{pair,11}_{\mathbb{1}_{x,t}=1}}{T^{pair,00}_{\mathbb{1}_{x,t}=1} + T^{pair,01}_{\mathbb{1}_{x,t}=1} + T^{pair,10}_{\mathbb{1}_{x,t}=1} + T^{pair,11}_{\mathbb{1}_{x,t}=1}}.$$

The likelihood ratio of Christoffersen (1998) is employed, i.e.

$$\begin{aligned} LR &= \left(\frac{\hat{p}}{\hat{p}_{01}}\right)^{T^{pair,01}_{\mathbb{1}_{x,t}=1}} \left(\frac{\hat{p}}{\hat{p}_{11}}\right)^{T^{pair,11}_{\mathbb{1}_{x,t}=1}} \\ &\quad \left(\frac{1-\hat{p}}{1-\hat{p}_{01}}\right)^{T^{pair,00}_{\mathbb{1}_{x,t}=1}} \left(\frac{1-\hat{p}}{1-\hat{p}_{11}}\right)^{T^{pair,10}_{\mathbb{1}_{x,t}=1}}, \end{aligned}$$

where $-2\log(LR) \sim \chi^2_1$. The frequency with which consecutive exceedances are observed may be few due to the fact that they are rare events, as a consequence the power of this test is limited.

## Appendix B. Bootstrap tests

In this section I briefly present the steps followed to build the boostrap tests. The main reason to build boostrap tests in estimated measures is due to the introduction of a nuisance parameter in the sample distribution. We estimate the model parameters to build the systemic measure and because of that, the distribution under the null hypothesis may be different, affecting to the confidence interval and the p-values. Durbin, J. (1973) points out the effect of estimated parameters in Kolgomorov Smirnov test. Abadie (2002) employs a boostrap procedure to build a

Kolgomorov Smirnov test when there are estimated parameters and Bernal et al. (2014) employs it in the $CoVaR$ framework. I extend the boostrap tests on $CoVaR$ framework to test a change in the mean.
The standard parametric t test assumes normality and homocedasticity in the sample distribution. In a boostrap procedure it is not necessary to make any assumption about the sample distribution, beyond the independence of the observations, because we do not use a theoretical probability distribution but the sample distribution under the null hypothesis. The following subsections show the steps in order to obtain the boostrap p-values.

*Appendix B.1. Boostrap t test*

Given two samples $x$ and $y$ with size $n_x$ and $n_y$:

Step 1 : Substract the mean for each sample and add the joint mean, i.e., $\tilde{x} = x - \bar{x} + \bar{z}$ and $\tilde{y} = y - \bar{y} + \bar{z}$ where $\bar{x}$ is the mean of the sample $x$, $\bar{y}$ is the mean of the sample $y$ and $\bar{z}$ is the mean of $z = [x; y]$.

Step 2 : Resample $n_x$ observations for $\tilde{x}$ and $n_y$ observations for $\tilde{y}$ obtaining two vector columns $x^b$ and $y^b$.

Step 3 : Asses t statistic

$$t^b = \frac{\bar{x}^b - \bar{y}^b}{\sqrt{\frac{\sigma^2_{x^b}}{n_x} + \sigma^2_{y^b} n_y}}$$

where $\bar{x}^b$ is the mean of

Step 4 Repeat steps Step 2 - Step 3 B times.

Step 5 Compare $t$ statistic from the original data, i.e., $t^{original}$, with the $t$ statistic from the simulated data, i.e. $t^b$ for $b = 1, ..., B$. Depending on the alternative hypothesis this last step is different (MacKinnon (2009)).

(A) $H_1 : \mu_x \neq \mu_y$

$$pvalue = 2 \min \left( \frac{\sum_{b=1}^{B} \mathbb{1}_{t^b > t^{original}} + 1}{B+1}, \frac{\sum_{b=1}^{B} \mathbb{1}_{t^b < t^{original}} + 1}{B+1} \right)$$

(B) $H_1 : \mu_x > \mu_y$

$$pvalue = \frac{\sum_{b=1}^{B} \mathbb{1}_{t^b > t^{original}} + 1}{B+1}$$

(C) $H_1 : \mu_x < \mu_y$

$$pvalue = \frac{\sum_{b=1}^{B} \mathbb{1}_{t^b < t^{original}} + 1}{B+1}$$

*Appendix B.2. Boostrap Kolgomorov Smirnov test*

These steps are obtained following Abadie (2002). Given two samples $x$ and $y$ with size $n_x$ and $n_y$:

Step 1 : Resample $N = n_x + n_y$ observations for $z = [x; y]$ obtaining a vector column $z^b$.

Step 2 : The first $n_x$ rows of column $z^b$ would be $x^b$ and the following $n_y$ would be $y^b$.

Step 3 : Assess $KS^{original}$ statistic or the modified version depending if your alternative hypothesis is $F_x(z) \neq F_y(z)$, i.e. not equal distribution, or $F_x(z) < F_y(z)$, i.e., first order stochastic dominance of $x$ over $y$.

Step 4 Repeat steps Step 1 - Step 3 B times.

Step 5 : Assess $KS^b$ statistic or the modified version for the original data

Step 6 : pvalues are obtained as:

$$pvalue = \frac{1 + \sum_{b=1}^{B} \mathbb{1}_{KS^b > KS^{original}}}{B + 1}$$

For the test where the alternative hypothesis is $H_1 : F_x(z) \neq F_y(z)$, the KS statistic is

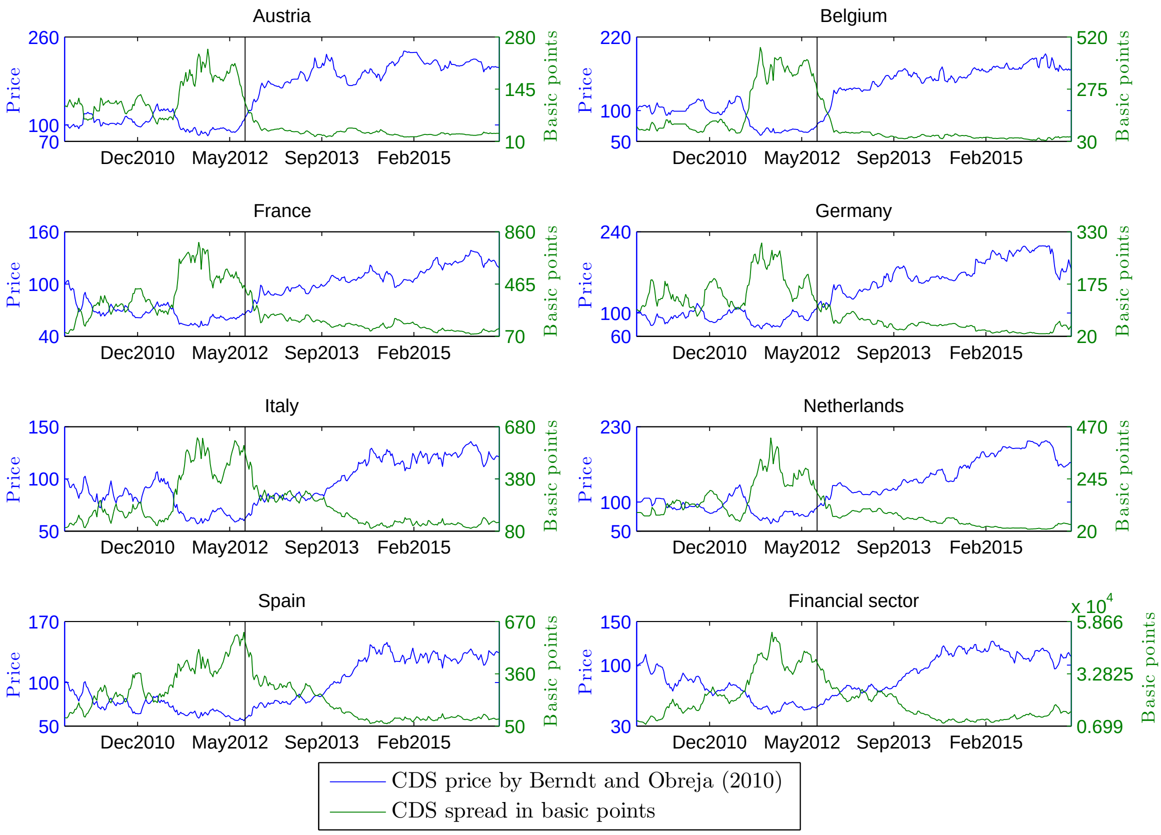
$$KS = \left( \frac{n_x n_y}{n_x + n_y} \right)^{1/2} sup_{z \in \mathbb{R}} \left| F_{x,n_x}(z) - F_{y,n_y}(z) \right|$$

and when the alternative hypothesis is $H_1 : F_x(z) < F_y(z)$, i.e., dominance of $x$ over $y$,

$$KS = \left( \frac{n_x n_y}{n_x + n_y} \right)^{1/2} sup_{z \in \mathbb{R}} \left( F_{x,n_x}(z) - F_{y,n_y}(z) \right).$$

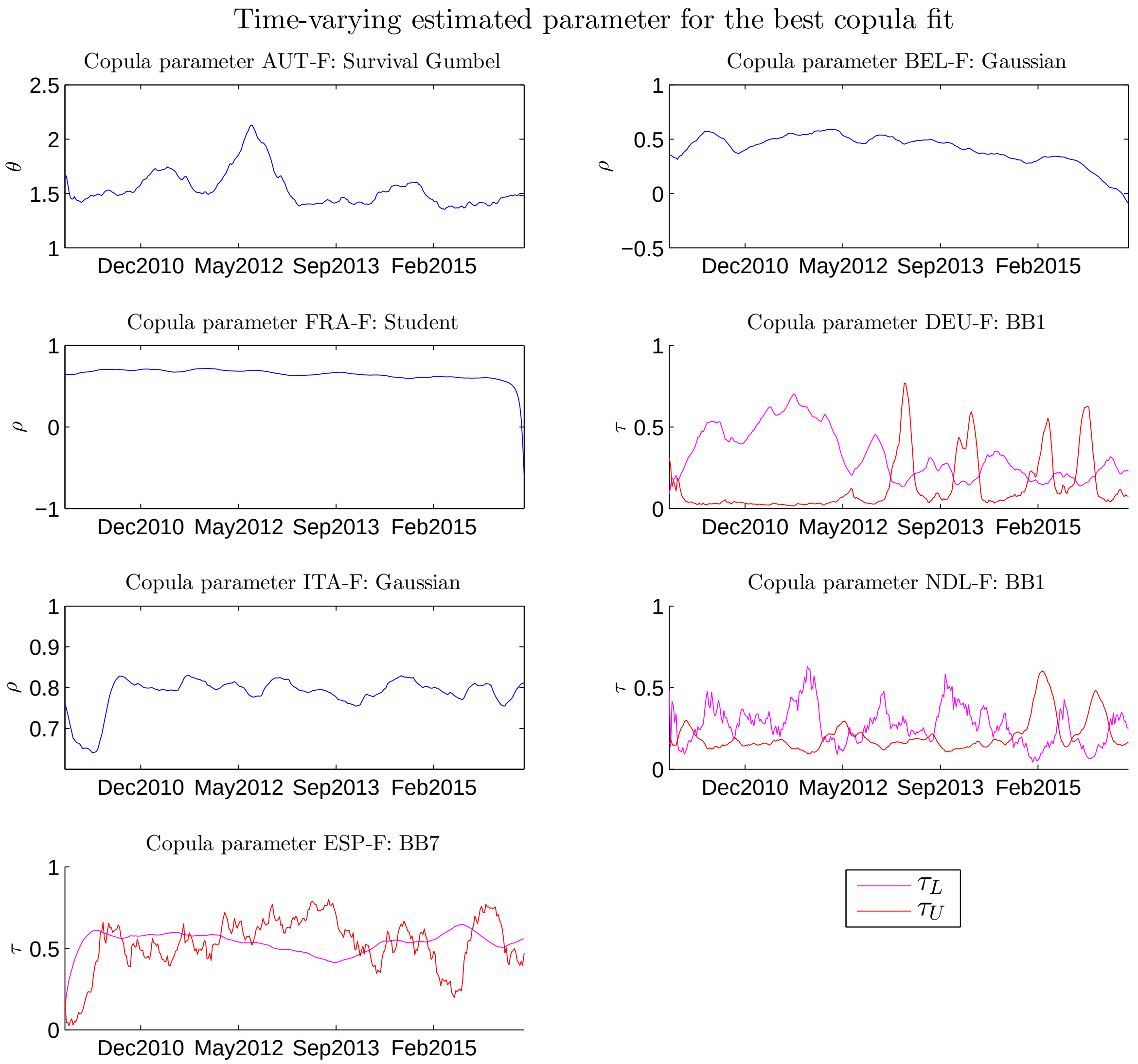
## Appendix C. Figures

Figure C.1: CDS spread and price following Berndt and Obreja (2010)



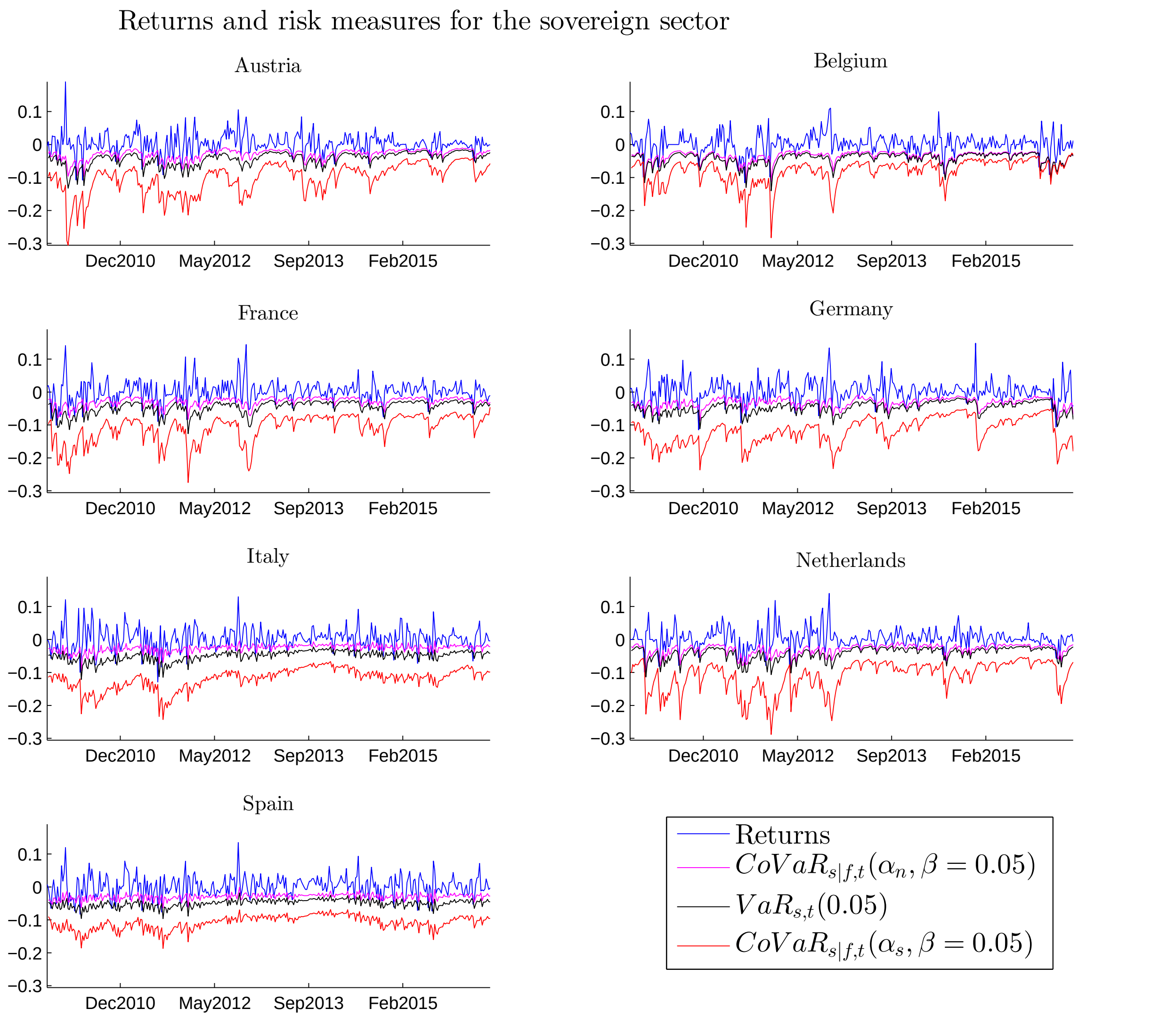

The green line represents CDS quotes. The blue line represents the price of an asset built using the exponential function of the returns from formula (11).
Both trends react with an opposite sign. The price seems to have a smoother path than the CDS quote. The value of time series is established in the initial date of the sample at 100. The vertical black line represents the week of July $26^{th}$, 2012, when Mario Draghi made the speech that changed the trend of the CDS quote for Spain and Italy.

Figure C.2: Time-varying evolution of the copula parameter



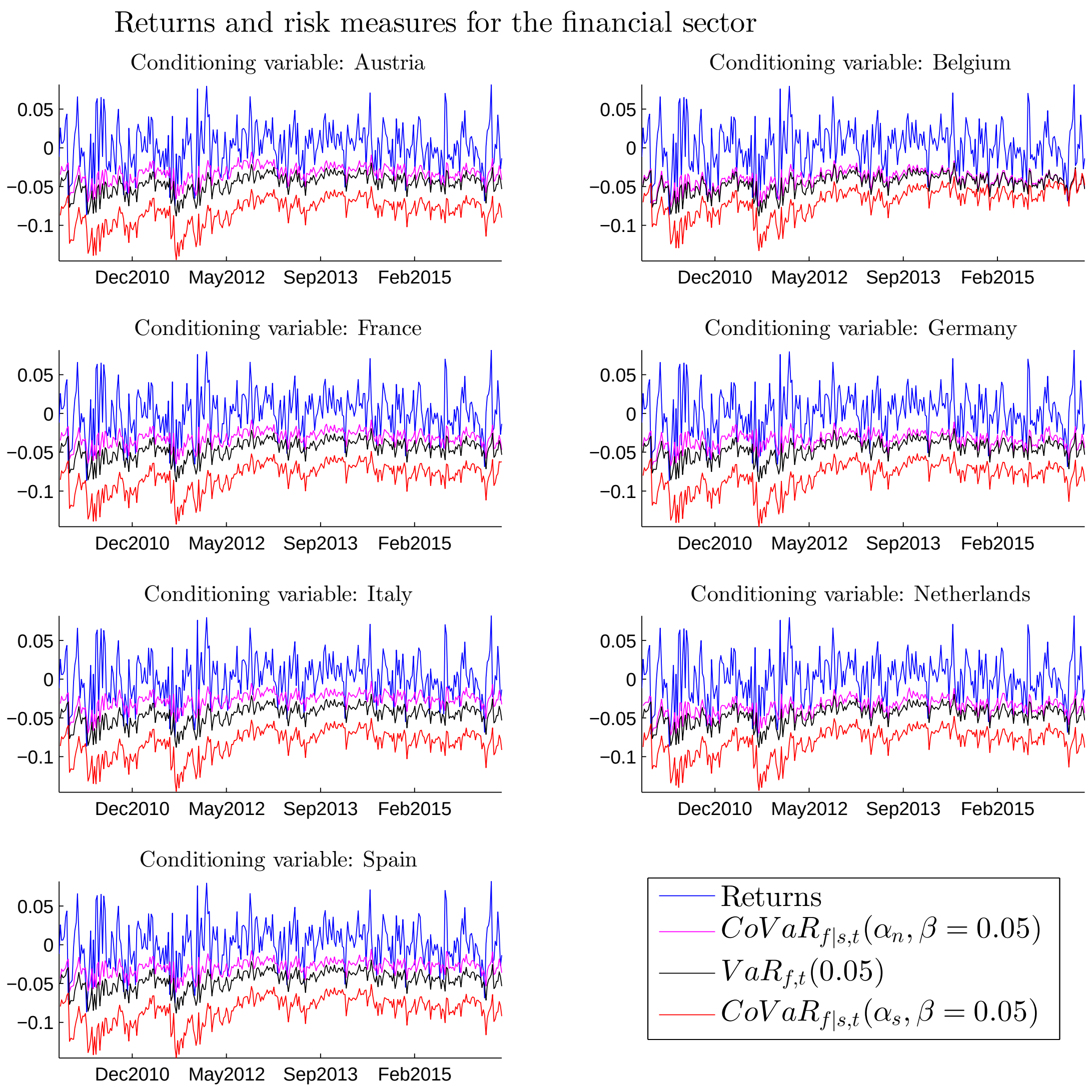

Time-varying evolution of the copula parameter for the best copula fit according to AICC. AUT: Austria. BEL : Belgium. FRAN: France. DEU: Germany. ITA: Italy. NDL: Netherlands. ESP: Spain. F. European financial sector.

Figure C.3: Weekly sovereign CDS returns and risk measures



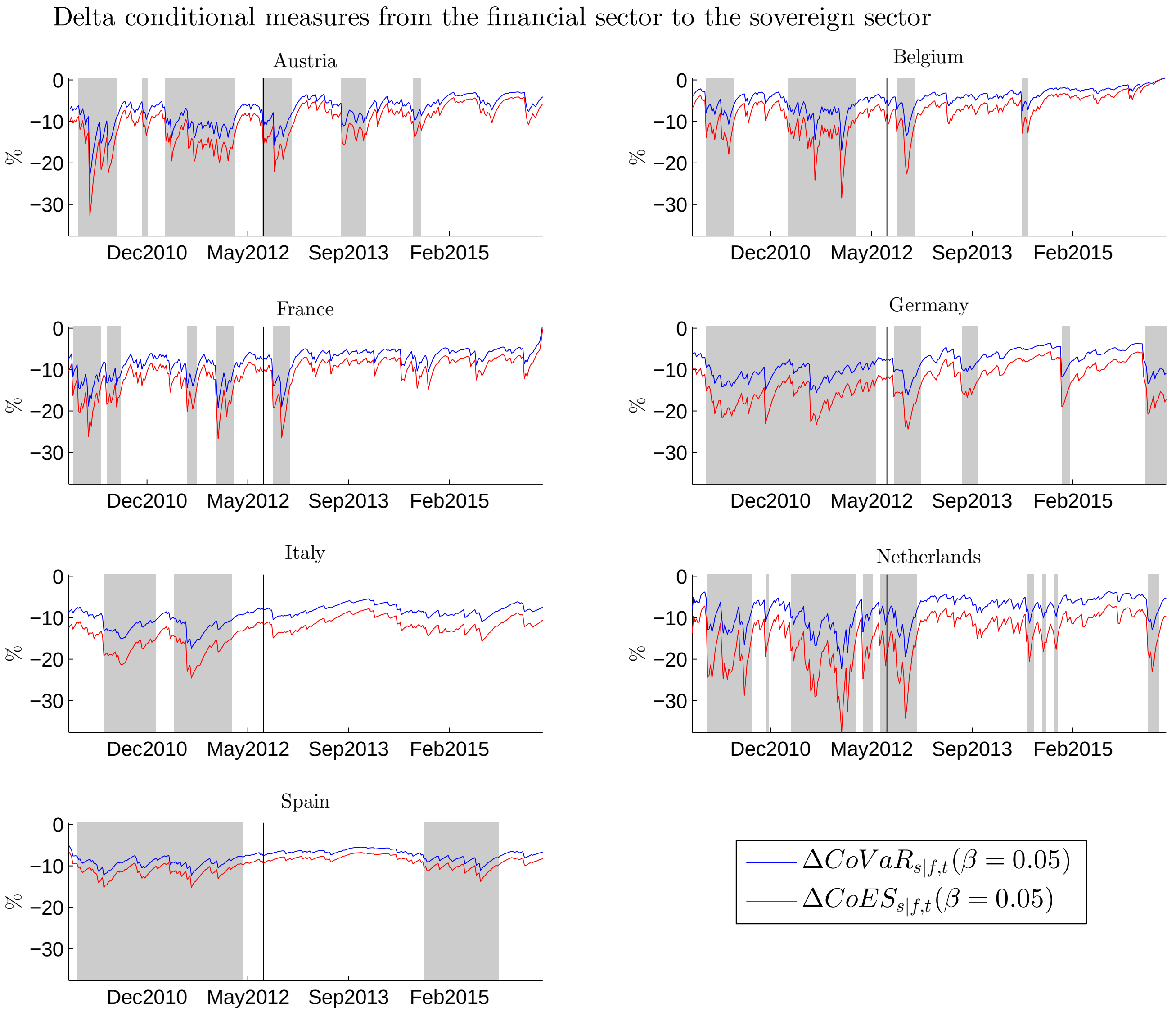

Time series plots for weekly sovereign CDS returns for the period 2009-2016 (in blue). Risk measures such as Value-at-Risk with a significance level of 5% (in black), $CoVaR$ for the sovereign sector with the same significance level when the financial sector is distress (in red) and when there are normal times for the financial sector (in magenta).

Figure C.4: Weekly CDS returns from financial sector and risk measures



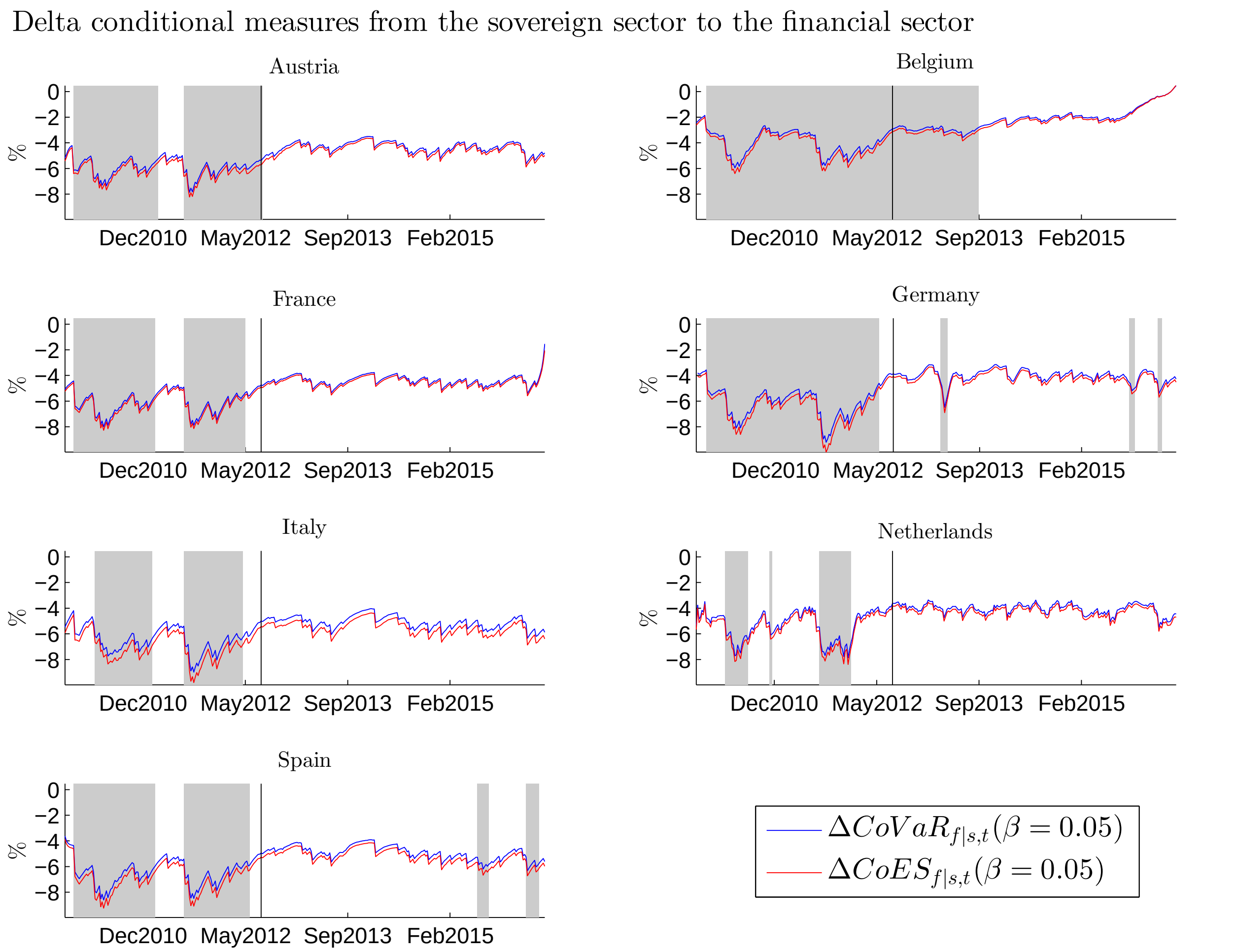

Time series plots for weekly CDS returns from financial sector for the period 2009-2016 (in blue). Risk measures such as Value-at-Risk with a significance level of 5% (in black), $CoVaR$ for the financial sector with the same significance level when the sovereign sector $s$ is distress (in red) and when there are normal times for the sovereign sector $s$ (in magenta).

Figure C.5: Time-varying evolution of the contagion measures from the financial sector to the sovereign sector




Delta conditional measures from the financial sector to the sovereign sector ($\Delta CoVaR_{s|f}(\beta = 0.05)$ and $\Delta CoES_{s|f}(\beta = 0.05)$ ) show the evolution of contagion from the financial sector to each European country. Distress periods (grey areas) are obtained from a Switching Markov model.

Figure C.6: Time-varying evolution of the contagion measures from the sovereign sector to the financial sector




Delta conditional measures from the sovereign sector to the financial sector ($\Delta CoVaR_{f|s}(\beta = 0.05)$ and $\Delta CoES_{f|s}(\beta = 0.05)$) show the evolution of contagion from each country to the European financial sector. Distress periods (grey areas) are obtained from a Switching Markov model.

## Appendix D. Tables

Table D.1: Main tail dependence features for each copula

| **Family** | **Lower tail dependence** | **Upper tail dependence** |
|---|---|---|
| Clayton | $2^{-1/\theta_t}$ | $-$ |
| Gumbel | $-$ | $2-2^{1/\theta_t}$ |
| Frank | $-$ | $-$ |
| BB7 (Joe-Clayton) | $2^{-1/\delta_t}$ | $2-2^{1/\theta_t}$ |
| Survival Gumbel | $2-2^{1/\theta_t}$ | $-$ |
| Student t | $2t_{\eta+1}\left(-\sqrt{\frac{(\eta+1)(1-\theta_t)}{1+\theta_t}}\right)$ | $2t_{\eta+1}\left(-\sqrt{\frac{(\eta+1)(1-\theta_t)}{1+\theta_t}}\right)$ |
| BB1 (Clayton-Gumbel) | $2^{-1/\theta_t\delta_t}$ | $2-2^{1/\delta_t}$ |
| Gaussian | $-$ | $-$ |

Note: $-$ represents that there is no tail dependency.
$\theta_t$ and $\delta_t$ are parameters of the copula at time $t$. The number of degrees of freedom of the Student t copula is $\eta$.
Source: (Ao et al., 2017, p. 22) and Jiang (2012).

Table D.2: Time-varying parameter representation for each copula

| *General model* | $\Lambda\left(\omega_K+\beta_K\theta_{t-1}^K+\alpha_K\frac{1}{20}\sum_{k=1}^{20}\lvert u_{s,t-k}-u_{f,t-k}\rvert\right)$ | |
|---|---|---|
| **Copula** | **Parameter** $\theta$ | **Function** $\Lambda(x)$ |
| Clayton | $\theta$ | $\exp(x)$ |
| Gumbel | $\theta$ | $(\exp(x)+1)$ |
| Frank | $\theta$ | $x$ |
| BB7 | $\tau^L$; $\tau^U$ | $(1+\exp(-x))^{-1}$ |
| Survival Gumbel | $\theta$ | $(\exp(x)+1)$ |
| Student t | $\rho$ | $\frac{1-\exp(-x)}{1+\exp(-x)}$ |
| BB1 | $\tau^U$: $\tau^L$ | $(1+\exp(-x))^{-1}$ |
| Gaussian | $\rho$ | $\frac{1-\exp(-x)}{1+\exp(-x)}$ |

Note:
$\tau^U, \tau^L \in (0,1)$.
For the BB7 copula $\theta = \frac{1}{\log_2(2-\tau^U)}$ and $\delta = \frac{-1}{\log_2(\tau^L)}$.
For the BB1 copula $\delta = \frac{1}{\log_2(2-\tau^U)}$ and $\theta = \frac{-\log_2(2-\tau^U)}{\log_2(\tau^L)}$.
The general model for elliptical copulas is: $\Lambda\left(\omega_K+\beta_K\theta_{t-1}^K+\alpha_K\frac{1}{20}\sum_{k=1}^{20}\Phi^{-1}(u_{s,t-k})\Phi^{-1}(u_{f,t-k})\right)$ where $\Phi^{-1}$ is the inverse Gaussian cumulative distribution function or the inverse Student t cumulative distribution function with $\eta$ degrees of freedom.

Table D.3: European banks employed for building the financial system credit risk index

| **Name** | **Country** |
|---|---|
| Banca Monte dei Paschi di Siena | Italy |
| Banco Comercial Português | Portugal |
| Banco Popular Español | Spain |
| Banco Santander | Spain |
| Bayerische Landesbk | Germany |
| BBVA | Spain |
| BNP Paribas | France |
| Commerzbank AG | Germany |
| Cooptieve Cente Rabo BA | Netherland |
| Credit Agricole | France |
| Credit Lyonnais | France |
| Danske Bank A/S | Finland |
| Deutsche bank AG | Germany |
| Erste Group Bank AG | Austria |
| ING Bank N.V. | Netherland |
| Intesa Sanpaolo Spa | Italy |
| KBCA Bank | Belgium |
| Lb Badenwuerttemberg | Germany |
| Mediobanca Spa | Italy |
| Natixis | France |
| Portigon AG | Germany |
| SNS Bank N.V. | Netherland |
| Societé Générale | France |
| Unicredit | Italy |
| Unicredit Bank AG | Austria |

Table D.4: Weights for building the financial sector proxy.

| Countries | $1^{st}$ PCA (%) |
|---|---|
| Austria | 10.83 |
| Belgium | 8.02 |
| Finland | 9.95 |
| France | 12.73 |
| Germany | 11.73 |
| Italy | 11.95 |
| Netherland | 12.62 |
| Portugal | 9.64 |
| Spain | 12.53 |

$1^{st}$ PCA column expresses the weights obtained by the first principal component.
Equal indicates the equally weighted portfolio.

Table D.5: Descriptive statistics: CDS returns.

| | Financial sector | Austria | Belgium | France | Germany | Italy | Netherlands | Spain |
|---|---|---|---|---|---|---|---|---|
| Mean | 0.0007 | 0.0026 | 0.0020 | 0.0010 | 0.0022 | 0.0012 | 0.0020 | 0.0014 |
| Maximum | 0.0818 | 0.1907 | 0.1102 | 0.1441 | 0.1482 | 0.1298 | 0.1398 | 0.1347 |
| Minimum | -0.0866 | -0.1081 | -0.1162 | -0.1008 | -0.1151 | -0.1292 | -0.0998 | -0.0826 |
| Std. Dev. | 0.0292 | 0.0310 | 0.0302 | 0.0324 | 0.0337 | 0.0338 | 0.0300 | 0.0317 |
| Skewness | -0.1031 | 0.4788 | -0.1247 | 0.3387 | 0.2664 | 0.1037 | 0.3422 | 0.2529 |
| Kurtosis | 3.2177 | 7.7882 | 5.3097 | 5.8765 | 5.1924 | 4.5488 | 5.8348 | 4.1274 |

Weekly data for the period May $22^{th}$, 2009 to May $13^{th}$, 2016. Returns obtained following Equation (11).

Table D.6: Estimates for the marginal distribution models.

| | Financial sector | Austria | Belgium | France | Germany | Italy | Netherlands | Spain |
|---|---|---|---|---|---|---|---|---|
| $\phi_0$ | 0.0007 | 0.0013 | 0.0019 | 0.0021 | 0.0017 | 0.0018 | 0.0009 | 0.0013 |
| | (0.44) | (1.05) | (1.42) | (1.39) | (1.12) | (1.05) | (0.88) | (0.77) |
| $\phi_1$ | 0.2542 | 0.2468 | 0.2418 | 0.2253 | 0.2603 | 0.2230 | 0.3035 | 0.2073 |
| | (4.80) | (4.27) | (4.14) | (3.76) | (4.61) | (4.18) | (19.27) | (3.89) |
| $\phi_2$ | - | - | - | - | - | -0.1515 | - | -0.1443 |
| | | | | | | (-2.91) | | (-2.68) |
| $\omega$ | 0.0000 | 0.0000 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0001 | 0.0000 |
| | (2.21) | (0.66) | (1.33) | (1.14) | (0.47) | (0.57) | (1.56) | (1.02) |
| $\alpha$ | 0.0000 | 0.2513 | 0.3164 | 0.2298 | 0.1208 | 0.0404 | 0.2297 | 0.0001 |
| | (0.00) | (2.38) | (1.93) | (1.83) | (1.03) | (1.07) | (4.23) | (0.00) |
| $\beta$ | 0.8974 | 0.7486 | 0.6342 | 0.7070 | 0.8577 | 0.9134 | 0.6996 | 0.8980 |
| | (70.62) | (7.57) | (4.43) | (4.75) | (8.60) | (11.36) | (12.92) | (11.42) |
| $\theta$ | 0.0956 | 0.0000 | 0.0006 | 0.0000 | 0.0430 | 0.0552 | 0.1414 | 0.1073 |
| | (1.79) | (0.00) | (0.00) | (0.00) | (0.35) | (0.62) | (6.25) | (1.33) |
| $\lambda$ | -0.0962 | 0.0367 | -0.0037 | 0.0111 | 0.0283 | -0.0169 | 0.0274 | -0.0750 |
| | (-1.22) | (0.58) | (-0.06) | (0.14) | (0.40) | (-0.23) | (1.05) | (-0.95) |
| $\eta$ | 27.9570 | 4.6061 | 3.8584 | 4.5905 | 3.9826 | 4.7036 | 3.2816 | 7.0903 |
| | (0.77) | (3.58) | (3.81) | (3.28) | (4.78) | (3.67) | (73.02) | (15.07) |
| *LogLike* | 736.398 | 745.540 | 755.618 | 728.359 | 707.992 | 698.789 | 767.387 | 705.898 |
| Kupiec (1995) | 0.598 | 0.317 | 0.830 | 0.598 | 0.444 | 0.144 | 0.444 | 0.969 |
| Christoffersen (1998) | 0.131 | 0.760 | 0.786 | 0.935 | 0.111 | 0.065 | 0.469 | 0.177 |
| LB | 0.550 | 0.352 | 0.824 | 0.561 | 0.970 | 0.882 | 0.139 | 0.948 |
| ARCH | 0.783 | 0.340 | 0.291 | 0.624 | 0.839 | 0.751 | 0.983 | 0.451 |

Notes: The table provides information on maximum likelihood parameter estimates and z-statistics (in brackets) for the marginal models in Equation (5)-(6). *LogLike* stands for the log-likelihood value. Kupiec (1995) and Christoffersen (1998) denote p-values of the unconditional coverage test from Kupiec (1995) and the conditional coverage test from Christoffersen (1998) with a significance level of 5%. LB and ARCH refer to p-values of the Ljung-Box test for serial correlation with 20 lags and the Engle's Lagrange multiplier for ARCH effects in the first lag.

Table D.7: Copula model estimates for financial and sovereign sectors' returns for the period 2009-2016. (I)

| | | Austria | Belgium | France | Germany | Italy | Netherlands | Spain |
|---|---|---|---|---|---|---|---|---|
| Clayton | $\omega$ | -0.33 | -0.78 | -0.40 | -0.41 | 1.72 | 1.07 | 2.59 |
| | | (0.15) | (0.23) | (0.32) | (0.25) | (0.35) | (0.92) | (1.09) |
| | $\alpha$ | -2.18 | -1.48 | -0.82 | -2.03 | -6.74 | -6.24 | -7.43 |
| | | (0.25) | (0.41) | (0.19) | (0.11) | (0.33) | (1.07) | (0.51) |
| | $\beta$ | 0.70 | 1.01 | 0.61 | 0.76 | -0.08 | 0.00 | -0.54 |
| | | (0.22) | (0.29) | (0.19) | (0.14) | (0.16) | (0.16) | (0.15) |
| Gumbel | $\omega$ | 1.95 | -2.27 | -1.99 | -2.82 | -0.60 | -0.01 | 0.53 |
| | | (0.35) | (0.13) | (0.67) | (2.69) | (0.13) | (0.79) | (0.13) |
| | $\alpha$ | -9.39 | -3.12 | -0.78 | -1.62 | -2.04 | 2.13 | -4.45 |
| | | (0.46) | (0.08) | (0.53) | (1.92) | (0.16) | (0.70) | (0.50) |
| | $\beta$ | -0.48 | 1.39 | 1.05 | 1.60 | 0.49 | -1.00 | 0.13 |
| | | (0.25) | (0.07) | (0.38) | (1.51) | (0.18) | (0.32) | (0.45) |
| Frank | $\omega$ | 1.24 | 0.41 | -0.28 | 0.36 | 1.96 | 3.95 | 8.95 |
| | | (0.35) | (0.33) | (0.01) | (20.13) | (0.06) | (1.06) | (0.35) |
| | $\alpha$ | -2.93 | -1.10 | 1.01 | -0.99 | -3.55 | 4.79 | -14.41 |
| | | (0.09) | (0.32) | (0.02) | (17.61) | (0.18) | (2.08) | (0.15) |
| | $\beta$ | 0.84 | 0.95 | 1.02 | 0.96 | 0.80 | -0.51 | -0.04 |
| | | (0.08) | (0.28) | (0.00) | (9.58) | (0.06) | (0.46) | (0.46) |
| BB7 | $\omega_L$ | 2.95 | 4.95 | -1.69 | 5.81 | -2.16 | 2.28 | -2.02 |
| | | (0.06) | (0.00) | (0.20) | (0.00) | (0.11) | (1.87) | (0.06) |
| | $\alpha_L$ | -12.06 | -20.30 | -1.04 | -24.28 | 0.59 | -13.31 | 0.48 |
| | | (0.01) | (0.05) | (0.14) | (0.01) | (0.08) | (0.06) | (0.09) |
| | $\beta_L$ | -2.59 | -3.95 | 3.77 | -4.07 | 4.14 | -0.19 | 3.90 |
| | | (0.01) | (0.08) | (0.12) | (0.05) | (0.00) | (1.29) | (0.02) |
| | $\omega_U$ | 0.77 | 1.78 | 0.60 | 2.74 | -0.78 | -3.20 | 2.66 |
| | | (0.08) | (0.01) | (0.05) | (0.01) | (0.12) | (1.44) | (0.18) |
| | $\alpha_U$ | -9.77 | -13.89 | -0.30 | -12.79 | -3.37 | 4.16 | -16.17 |
| | | (0.08) | (0.07) | (0.21) | (0.01) | (0.10) | (0.46) | (0.02) |
| | $\beta_U$ | 1.13 | -1.68 | -2.12 | -5.81 | 2.78 | 4.04 | 0.41 |
| | | (0.02) | (0.01) | (0.04) | (0.06) | (0.1) | (1.18) | (0.00) |

Notes: The table provides information on maximum likelihood parameter estimates and standard deviation (in brackets) for the copula models in Equations (8), (9) and (10). Standard deviation of copula parameters has been computed following the sandwich form presented in Patton (2013).

Table D.8: Copula model estimates for financial and sovereign sectors' returns for the period 2009-2016. (II)

| | | Austria | Belgium | France | Germany | Italy | Netherlands | Spain |
|---|---|---|---|---|---|---|---|---|
| Survival Gumbel | $\omega$ | -2.23 | -0.70 | -2.05 | -2.28 | 0.60 | -0.31 | 2.96 |
| | | (0.03) | (0.01) | (0.01) | (0.16) | (0.08) | (0.01) | (0.02) |
| | $\alpha$ | -1.57 | -5.12 | -0.58 | -2.26 | -4.75 | -3.33 | -7.27 |
| | | (0.02) | (0.03) | (0.01) | (0.01) | (0.44) | (0.01) | (0.03) |
| | $\beta$ | 1.24 | 0.63 | 1.07 | 1.35 | 0.13 | 0.15 | -0.80 |
| | | (0.06) | (0.02) | (0.01) | (0.01) | (1.07) | (0.02) | (0.03) |
| Student t | $\eta$ | 9.46 | 19.69 | 4.16 | 10.73 | 99.67 | 41.62 | 11.33 |
| | | (0.21) | (0.00) | (0.01) | (0.00) | (0.13) | (0.00) | (0.00) |
| | $\omega$ | 1.42 | 1.39 | -0.53 | 2.27 | 1.25 | 1.35 | 1.93 |
| | | (0.31) | (0.00) | (0.01) | (0.01) | (0.04) | (0.01) | (0.06) |
| | $\alpha$ | 0.64 | -2.04 | -0.37 | -4.36 | 8.79 | -0.98 | 3.55 |
| | | (0.24) | (0.23) | (0.03) | (0.01) | (0.19) | (0.02) | (0.02) |
| | $\beta$ | -0.98 | 1.08 | 3.47 | 1.73 | -4.31 | 0.34 | -2.27 |
| | | (0.10) | (0.12) | (0.02) | (0.01) | (0.16) | (0.00) | (0.02) |
| BB1 | $\omega_L$ | 2.35 | 6.36 | -1.77 | -1.12 | 0.66 | 2.74 | 1.80 |
| | | (0.12) | (0.2) | (0.01) | (0.03) | (0.02) | (0.02) | (0.01) |
| | $\alpha_L$ | -10.90 | -28.76 | -0.96 | -3.64 | -6.83 | -16.46 | -4.95 |
| | | (0.12) | (0.20) | (0.01) | (0.03) | (0.11) | (0.01) | (0.01) |
| | $\beta_L$ | -2.90 | -4.93 | 3.88 | 3.56 | 0.79 | -0.34 | -2.33 |
| | | (0.01) | (0.12) | (0.02) | (0.01) | (0.03) | (0.02) | (0.02) |
| | $\omega_U$ | -0.60 | 1.10 | -0.86 | -5.74 | -1.96 | -3.12 | 0.35 |
| | | (0.07) | (0.08) | (0.01) | (0.02) | (0.03) | (0.01) | (0 (0.02) |
| | $\alpha_U$ | -5.27 | -7.68 | 1.77 | 11.97 | 0.02 | 3.79 | -5.86 |
| | | (0.01) | (0.07) | (0.00) | (0.04) | (0.00) | (0.01) | (0.04) |
| | $\beta_U$ | 2.36 | -8.55 | -0.39 | 4.32 | 3.92 | 4.04 | 1.19 |
| | | (0.01) | (0.04) | (0.01) | (0.02) | (0.07) | (0.01) | (0.00) |
| Gaussian | $\omega$ | -0.01 | -0.05 | 2.24 | 0.16 | -1.42 | 0.46 | -0.33 |
| | | (0.03) | (0.07) | (0.08) | (0.12) | (0.03) | (0.04) | (0.02) |
| | $\alpha$ | 0.20 | 0.13 | 0.87 | 0.25 | 0.11 | 0.17 | 0.28 |
| | | (0.01) | (0.09) | (0.09) | (0.02) | (0.04) | (0.01) | (0.01) |
| | $\beta$ | 2.08 | 2.16 | -1.90 | 1.61 | 4.42 | 1.03 | 2.78 |
| | | (0.01) | (0.04) | (0.01) | (0.08) | (0.06) | (0.05) | (0.01) |

Notes: The table provides information on maximum likelihood parameter estimates and standard deviation (in brackets) for the copula models in Equations (8), (9) and (10). Standard deviation of copula parameters has been computed following the sandwich form presented in Patton (2013).

Table D.9: Value of the Akaike Information Criterion corrected for small sample bias for the considered copulas.

| | Austria | Belgium | France | Germany | Italy | Netherlands | Spain |
|---|---|---|---|---|---|---|---|
| *AICC* | | | | | | | |
| Clayton | -106.67 | -64.46 | -170.21 | -88.85 | -242.52 | -79.10 | -216.72 |
| Gumbel | -100.29 | -47.99 | -169.62 | -74.43 | -279.59 | -66.11 | -256.65 |
| Frank | -107.79 | -59.59 | -172.07 | -84.69 | -270.14 | -83.53 | -231.55 |
| BB7 | -113.66 | -64.68 | -195.68 | -94.36 | -304.67 | -82.87 | **-282.81** |
| Survival Gumbel | **-117.45** | -65.21 | -193.20 | -96.62 | -288.21 | -84.55 | -260.46 |
| Student t | -111.81 | -56.65 | **-199.06** | -95.09 | -306.85 | -81.44 | -268.17 |
| BB1 | -115.02 | -67.96 | -196.52 | **-99.25** | -301.78 | **-84.74** | -273.48 |
| Gaussian | -116.39 | **-68.54** | -184.78 | -91.99 | **-307.65** | -83.77 | -273.13 |

Notes: $AICC$ denotes Akaike Information Criterion corrected for small sample bias. $AICC = 2k\frac{T}{T-k-1} - 2\log(\hat{L})$ where $T$ is the sample size, $k$ is the number of estimated parameters and $\hat{L}$ is the Log-likelihood value. Minimum $AICC$ value (in bold) indicates the best copula fit.

Table D.10: CoVaR backtesting

| | | | Austria | Belgium | France | Germany | Italy | Netherlands | Spain |
|---|---|---|---|---|---|---|---|---|---|
| $CoVaR_{f\|s,t}(0.5, 0.05)$ | Kupiek | pvalue | 0.5635 | 0.8319 | 0.4980 | 0.5635 | 0.5750 | 0.3209 | 0.7244 |
| | | Lower bound | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| | | Upper bound | 14 | 14 | 15 | 14 | 14 | 14 | 13 |
| | | # exceedances | 7 | 8 | 7 | 7 | 7 | 6 | 7 |
| | | # observations | 172 | 172 | 178 | 172 | 171 | 174 | 159 |
| | Christoffersen | pvalue | 0.4394 | 0.3754 | 0.4476 | 0.4394 | 0.4380 | 0.5114 | 0.4204 |
| | | $T_{00}$ | 157 | 155 | 163 | 157 | 156 | 161 | 144 |
| | | $T_{01}$ | 7 | 8 | 7 | 7 | 7 | 6 | 7 |
| | | $T_{10}$ | 7 | 8 | 7 | 7 | 7 | 6 | 7 |
| | | $T_{11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $CoVaR_{s\|f,t}(0.5, 0.05)$ | Kupiek | pvalue | 0.4095 | 0.8454 | 0.1456 | 0.8755 | 0.8755 | 0.8454 | 0.5750 |
| | | Lower bound | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| | | Upper bound | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| | | # exceedances | 11 | 8 | 13 | 9 | 9 | 8 | 7 |
| | | # observations | 171 | 171 | 171 | 171 | 171 | 171 | 171 |
| | Christoffersen | pvalue | 0.2171 | 0.3740 | 0.1421 | 0.3157 | 0.3157 | 0.3740 | 0.4380 |
| | | $T_{00}$ | 148 | 154 | 144 | 152 | 152 | 154 | 156 |
| | | $T_{01}$ | 11 | 8 | 13 | 9 | 9 | 8 | 7 |
| | | $T_{10}$ | 11 | 8 | 13 | 9 | 9 | 8 | 7 |
| | | $T_{11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Kupiec refers to the unconditional coverage test from Kupiec (1995) whereas Christoffersen denotes the conditional coverage test from Christoffersen (1998).

Backtesting is drawing up employing observations where returns of the conditioning variable are lower than the minimum return according to $VaR_{x,t}(0.5)$. The number of observations that cross the threshold is showed in *# observations*. Given these observations, the backtesting for the conditioned variable is computed using those returns below the $CoVaR_{y|x,t}(0.5, 0.05)$.

Confidence interval for the null hypothesis is presented in the upper bound and lower bound rows. The actual number of exceedances is presented in *# exceedances*.

For the conditional coverage test by Christoffersen (1998) about $CoVaR$, $T_{00}$ indicates the number of pairs of observation where no exceedance occurs neither in $t-1$ nor in $t$, $T_{11}$ shows the number of pairs of observation where an exceedance occurs in $t-1$ and in $t$, $T_{01}$ shows the number of pairs of observation where an exceedance occurs in $t$ but no in $t-1$, and $T_{10}$ indicates the number of pairs of observation where an exceedance occurs in $t-1$ but no in $t$.

Table D.11: Boostrap pvalues

| Country-Financial sector | Measure | $t1$ | $t2$ | $KS1$ | $KS2$ |
|---|---|---|---|---|---|
| Austria | F $\rightarrow$ S | 0.0276 | 0.0135 | 0.0226 | 0.0099 |
| | S $\rightarrow$ F | 0.0104 | 0.0047 | 0.1372 | 0.3018 |
| Belgium | F $\rightarrow$ S | 0.0514 | 0.0264 | 0.0018 | 0.0011 |
| | S $\rightarrow$ F | 0.1770 | 0.0811 | 0.0002 | 0.0002 |
| France | F $\rightarrow$ S | 0.0832 | 0.0358 | 0.0014 | 0.0006 |
| | S $\rightarrow$ F | 0.1608 | 0.0796 | 0.0017 | 0.0007 |
| Germany | F $\rightarrow$ S | 0.0884 | 0.0427 | 0.0354 | 0.6281 |
| | S $\rightarrow$ F | 0.0178 | 0.0102 | 0.5345 | 0.3739 |
| Italy | F $\rightarrow$ S | 0.1202 | 0.0584 | 0.0014 | 0.0008 |
| | S $\rightarrow$ F | 0.0352 | 0.0188 | 0.0146 | 0.0081 |
| Netherlands | F $\rightarrow$ S | 0.0160 | 0.0110 | 0.0821 | 0.0377 |
| | S $\rightarrow$ F | 0.1022 | 0.0521 | 0.0536 | 0.0626 |
| Spain | F $\rightarrow$ S | 0.0878 | 0.0451 | 0.0039 | 0.0020 |
| | S $\rightarrow$ F | 0.0280 | 0.0138 | 0.0006 | 0.0004 |

F $\rightarrow$ S stands for the orthogonalized $-\Delta CoVaR_{s|f,t}(0.05)$ whereas S $\rightarrow$ F indicates the orthogonalized $-\Delta CoVaR_{f|s,t}(0.05)$. $\Delta CoVaR_{f|s,t}(0.05)$ is multiplied by minus one in order to speak about levels of contagion, and due to orthogonalization the $\Delta CoVaR$ is not explained by its previous month. The chosen breakpoint is July $26^{th}$, 2012. All p-values are obtained using a boostrap procedure explained in Appendix B using $B = 10000$ simulations.
$t1$ shows the p-vale of t test where the null hypothesis is that the mean of level of contagion not explained by the previous month is the same before and after the breakpoint, i.e. $H_0 : \mu_B = \mu_A$ and $H_1 : \mu_B \neq \mu_A$ whereas the alternative hypothesis in $t2$ is $H_1 : \mu_B > \mu_A$.
$KS1$ shows the p-value of the Kolgomorov Smirnov test where the null and alternative hypothesis are $H_0 : F_B(z) = F_A(z) \forall z$ and $H_0 : F_B(z) \neq F_A(z) \forall z$ where $B$ is the level of contagion not explained by the previous month before July $26^{th}$, 2012 and $A$ is the same variable after that date.
$KS2$ indicates the p-value to the Kolgomorov Smirnov test where the alternative hypothesis is the first-order stochastic dominance of the distribution of the contagion before the breakpoint over the distribution of the contagion after the breakpoint, i.e., $H_1 : F_B(z) < F_A(z) \forall z$. In order words, for any level of contagion $z$ it would be a more extreme scenario after that before the breakpoint.